Differential Signal and Common Mode Signal in Time Domain

Most of multi-Gbps IO technologies use differential signaling, and their typical signal path impedance is 100ohm differential. Two 50ohm cables, however, are usually used in parallel to connect a DUT to/from test and measurement instruments such as oscilloscope and BERT. One may wonder why two 50ohm cables in parallel make 100ohm differential path. Or, one may wonder why it is often said that the two cables’ characteristic such as cable type and length must be matched well. Starting with the definition of differential signal, we will discuss 100ohm differential signal path vs. dual 50ohm signal path in detail in this article. In the end, we will study why the two 50ohm cables’ length must be matched well when they are used to send differential signal.

1.0 Mode of Signal

1.1 Differential Mode and Common Mode

Most of practical high speed IOs use two signal lines plus ground to send differential signal. Such a transmission medium is considered as a four-port network. The voltages and the currents at the four ports are denoted here as shown in Fig.1.

![Fig.1 A Network with Two Input and Two Output Ports](image)

Assuming that $v_1$ and $v_2$ are input voltages and $i_1$ and $i_2$ are input currents, the differential voltage, the common mode voltage, the differential current and the common mode current are defined by (Eq.1) through (Eq.4) respectively.

\[
\begin{align*}
  v_d &= v_1 - v_2 \\
  v_c &= \frac{v_1 + v_2}{2} \\
  i_d &= \frac{i_1 - i_2}{2} \\
  i_c &= i_1 + i_2
\end{align*}
\]  

(Eq.1)  
(Eq.2)  
(Eq.3)  
(Eq.4)

While (Eq.1) and (Eq.2) are intuitively understandable as the two voltages’ difference and the average respectively, one may wonder why the differential current is the two currents’ difference divided by two, and the common mode current is the sum of the two currents. The reason is because the total power calculated with $(v_1, i_1, v_2, \text{ and } i_2)$ must be equal to the total power calculated with $(v_d, i_d, v_c, \text{ and } i_c)$ as expressed by (Eq.5) and (Eq.6).
1.2 Odd Mode and Even Mode

Defining even and odd voltage vectors by (Eq.7) and even and odd current vectors by (Eq.8), $v_1$ and $v_2$ are expressed by (Eq.9), and $i_1$ and $i_2$ are expressed by (Eq.10). From (Eq.9), $v_c$ and $v_d$ can be considered as the coefficients of even voltage vector and odd voltage vector respectively. From (Eq.10), $i_c$ and $i_d$ can be considered as the coefficients of even current vector and odd current vector respectively.

\[
\begin{align*}
\vec{v}_{even} &= \frac{1}{1} \\
\vec{v}_{odd} &= \frac{1}{-1}
\end{align*}
\]  
(Eq.7)

\[
\begin{align*}
\vec{i}_{even} &= \frac{1}{1} \\
\vec{i}_{odd} &= \frac{1}{-1}
\end{align*}
\]  
(Eq.8)

\[
\begin{align*}
\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} &= \begin{bmatrix} v_c \\ v_d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\
\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} &= \begin{bmatrix} i_c \\ -i_d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{align*}
\]  
(Eq.9)

\[
\begin{align*}
\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} &= \begin{bmatrix} v_c \\ v_d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\
\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} &= \begin{bmatrix} i_c \\ -i_d \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\end{align*}
\]  
(Eq.10)

The electric fields of the even and odd modes of a coupled microstrip lines are illustrated in Fig.2.

![Fig.2 E-field Cross Sections of Even and Odd Modes of a Coupled Microstrip Lines](image)

(a) Even Mode  (b) Odd Mode

Although even and odd modes may seem to be introduced only for convenience in the discussion above, these two modes are the eigenvectors of the circuit (i.e., there is a significant physical meaning) when the signal path consisting of two signal lines is symmetric. Refer to the subsection 3.1.
2.0 Impedance and Signal Path

Having defined differential and common mode voltages and currents, let’s study the impedance of each mode.

2.1 Geometrical Characteristic of Signal Path

2.1.1 Asymmetric Signal Path

The voltage and current pairs at the input of an asymmetric signal path is related via an impedance matrix as expressed by (Eq.11). To obtain the common mode impedance, force even mode current, then \(v_1\) and \(v_2\) are measured as expressed by (Eq.12). From the definition of common mode voltage and current, (Eq.13) and (Eq.14) are obtained, from which the common mode impedance is obtained as expressed by (Eq.15). Forcing odd mode current, the differential mode impedance is obtained likewise as expressed by (Eq.19).

\[
\begin{bmatrix}
  v_1 \\
  v_2
\end{bmatrix} =
\begin{bmatrix}
  Z_{11} & Z_{12} \\
  Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2
\end{bmatrix}
\]

For even mode current \(i_1 = i_2 = i_0\)

\[
v_1 = (Z_{11} + Z_{12})i_0
\]

\[
v_2 = (Z_{21} + Z_{22})i_0
\]

\[
\begin{align*}
v_c &= \frac{v_1 + v_2}{2} = \frac{Z_{11} + Z_{12} + Z_{21} + Z_{22}}{2}i_0 \\
i_c &= i_1 + i_2 = 2i_0
\end{align*}
\]

\[
\therefore Z_c = \frac{v_c}{i_c} = \frac{Z_{11} + Z_{12} + Z_{21} + Z_{22}}{4}
\]

For odd mode current \(i_1 = -i_2 = i_0\)

\[
v_1 = (Z_{11} - Z_{12})i_0
\]

\[
v_2 = (Z_{21} - Z_{22})i_0
\]

\[
\begin{align*}
v_d &= v_1 - v_2 = \left[(Z_{11} + Z_{22}) - (Z_{12} + Z_{21})\right]i_0 \\
i_d &= i_1 - i_2 = i_0
\end{align*}
\]

\[
\therefore Z_d = \frac{v_d}{i_d} = (Z_{11} + Z_{22}) - (Z_{12} + Z_{21})
\]

2.1.2 Symmetric Signal Path

Symmetric signal path is used for differential signal in practice, where the voltage and current relation is expressed by (Eq.20). Since \(Z_a\) is the single-ended impedance when only one signal line is excited, and \(Z_b\) is related to the coupling of the two lines, let’s rename them as expressed by (Eq.21). Then the voltage and current relation is expressed by (Eq.22). Forcing even mode current, the common mode impedance is obtained as expressed by (Eq.23). Forcing odd mode current, the differential mode impedance is obtained as expressed by (Eq.24).
When even mode current is forced, the even mode voltage is obtained as expressed by (Eq.25), and the even mode impedance is obtained as expressed by (Eq.26). When odd mode current is forced, the odd mode voltage is obtained as expressed by (Eq.27), and the odd mode impedance is obtained as expressed by (Eq.28). From (Eq.23) and (Eq.26), the relation between the common mode impedance and the even mode impedance is obtained as expressed by (Eq.29). From (Eq.24) and (Eq.28), the relation between the differential mode impedance and the odd mode impedance is obtained as expressed by (Eq.30).

\[
\begin{align*}
\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} &= \begin{bmatrix} Z_a & Z_b \\ Z_b & Z_a \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \\
Z_a &= Z_0 \\
Z_b &= \alpha \times Z_0 \quad (\alpha \geq 0)
\end{align*}
\]  
(Eq.20)

\[
\begin{align*}
\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} &= \begin{bmatrix} Z_0 & \alpha Z_0 \\ \alpha Z_0 & Z_0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \\
Z_e &= \frac{v_c}{i_c} = \frac{1}{2} Z_0 (1 + \alpha) \\
Z_d &= \frac{v_d}{i_d} = 2 Z_0 (1 - \alpha)
\end{align*}
\]  
(Eq.22)

\[
\begin{align*}
\begin{bmatrix} v_{even} \\ v_{odd} \end{bmatrix} &= \begin{bmatrix} Z_0 & 0 \\ 0 & Z_0 \end{bmatrix} \begin{bmatrix} i_{even} \\ i_{odd} \end{bmatrix} \\
\Rightarrow Z_{even} &= \frac{v_{even}}{i_{even}} = Z_0 (1 + \alpha) \geq Z_0 \\
\Rightarrow Z_{odd} &= \frac{v_{odd}}{i_{odd}} = Z_0 (1 - \alpha) \leq Z_0
\end{align*}
\]  
(Eq.26)

When two 50ohm cables are used in parallel, there is no coupling between the two cables. Therefore, the odd mode impedance of this dual 50ohm cables path becomes 50ohm by (Eq.28). Then the differential impedance becomes 100ohm by (Eq.30).
2.2 Impedance Matching at Receiver

With differential signaling, our intention is usually such that differential mode is used to send useful information and common mode is used to provide DC bias. In this case, signal path termination is considered only for differential signal, that is, only differential impedance matching is considered. AC common mode, however, usually exists in real life. In this case, if common mode impedance is not matched, AC common mode is reflected, and it could cause problems such as generating differential noise through mode conversion and EMI. Let’s analyze differential and common mode impedance matching with the following three examples. Note that differential impedance matching is equivalent to odd mode impedance matching, and common mode impedance matching is equivalent to even mode impedance matching. We use even/odd modes impedance matching here for simplicity.

2.2.1 LVDS Type

LVDS type termination scheme is shown in Fig.3. The even mode impedance at the device input is infinite. If AC even mode exits, it is 100% reflected at the device input. The odd mode impedance matching is achieved by matching \( R_a/2 \) to the odd mode impedance of the signal path.

\[
Z_{\text{even}} = \infty \\
Z_{\text{odd}} = \frac{R_a}{2}
\]

Fig.3 LVDS Type Termination

In order to achieve impedance matching for both odd mode and even mode, two more resistors \((R_b \times 2)\) are required as shown in Fig.4. Since the even mode signal does not see \( R_a \), and \( R_b \) directly terminates the even mode signal, \( R_b \) should be equal to the even mode impedance of the signal path. Since the odd mode signal sees \( R_a/2 \) and \( R_b \) in parallel, the odd mode impedance matching condition is expressed by (Eq.31), from which \( R_a \) is obtained as expressed by (Eq.32).

\[
\begin{align*}
(1) \text{ even mode impedance matching} \\
R_b &= Z_{\text{even}}
\end{align*}
\]

\[
\begin{align*}
(2) \text{ odd mode impedance matching} \\
\frac{R_a}{2} &\parallel Z_{\text{odd}} \\
\frac{Z_{\text{even}} \cdot R_a}{2Z_{\text{even}} + R_a} &= Z_{\text{odd}} \\
\therefore R_a &= \frac{2Z_{\text{even}} \cdot Z_{\text{odd}}}{Z_{\text{even}} - Z_{\text{odd}}} \\
\end{align*}
\]

Fig.4 Modified LVDS Type Termination
2.2.2 CML Type

CML type termination is shown in Fig.5. Both the even and the odd mode signals directly see \( R_a \) at the device input. When there is no coupling between the two symmetric signal lines, the even mode impedance and the odd mode impedance become the same as expressed by (Eq.26) and (Eq.28). In this case, impedance matching for both odd mode and even mode is achieved by matching \( R_a \) to the even/odd mode signal path impedance.

\[
Z_{even} = R_a \\
Z_{odd} = R_a
\]

Fig.5 CML Type Termination

When the even mode impedance and the odd mode impedance are different, one more resistor \( (R_b) \) is required to achieve the impedance matching for both modes as shown in Fig.6. Since the odd mode signal directly sees \( R_a \) at the device input, \( R_a \) needs to be equal to the odd mode signal path impedance. The even mode impedance at the device input is expressed by (Eq.33), from which \( R_b \) is obtained as expressed by (Eq.34).

\[
R_a = Z_{odd} \\
\frac{v_e}{i_e} = \frac{Z_{odd} + 2R_b}{i_e} \\
Z_{even} = \frac{v_e}{i_e} = Z_{odd} + 2R_b \quad \text{(Eq.33)} \\
\therefore R_b = \frac{Z_{even} - Z_{odd}}{2} \quad \text{(Eq.34)}
\]

Fig.6 Modified CML Type Termination

2.2.3 Test and Measurement Instruments

Typical outputs or inputs terminals of test and measurement instruments are two 50ohm connectors, one for true signal and another for complement signal. Utilizing CML type termination, impedance matching for both differential mode and common mode is achieved.


3.0 Mode Conversion

Assume that useful information is sent by differential mode signal (i.e. odd mode signal) at a transmitter. Ideal situation is that the differential signal propagates to a receiver without distortion. Part of the original differential signal, however, could be converted to common mode signal during propagation, and the receiver might not fully receive the original signal.

3.1 Condition for No Mode Conversion

Let’s study the condition under which mode conversion does not occur. Then we will know when mode conversion would occur too. Applying even mode current to (Eq.11), the resulting differential voltage is obtained by (Eq.35). Applying odd mode current to (Eq.11), the resulting common mode voltage is obtained by (Eq.36).

\[
\begin{align*}
\text{For even mode current} & \quad i_1 = i_2 = i_0 \\
v_d &= v_1 - v_2 = [(Z_{11} + Z_{12}) - (Z_{21} + Z_{22})] \times i_0 \quad \text{(Eq.35)}
\end{align*}
\]

\[
\begin{align*}
\text{For odd mode current} & \quad i_1 = -i_2 = i_0 \\
v_c &= \frac{v_1 + v_2}{2} = \frac{(Z_{11} - Z_{12}) + (Z_{21} - Z_{22})}{2} \times i_0 \quad \text{(Eq.36)}
\end{align*}
\]

In order for mode conversion not to occur, the differential mode voltage expressed by (Eq.35) and the common mode voltage expressed by (Eq.36) must be zero. Then the no-mode-conversion condition is obtained as expressed by (Eq.39).

\[
\begin{align*}
\text{For no mode conversion to occur,} \\
\begin{cases}
(Z_{11} + Z_{12}) - (Z_{21} + Z_{22}) = 0 \\
(Z_{11} - Z_{12}) + (Z_{21} - Z_{22}) = 0
\end{cases} \quad \text{(Eq.37)}
\end{align*}
\]

\[
\begin{align*}
\text{Equivalent to} & \quad Z_{12} = Z_{21} \text{ and } Z_{11} = Z_{22} \quad \text{(Eq.39)}
\end{align*}
\]

Therefore the impedance matrix needs to be symmetric for mode conversion not to occur as expressed by (Eq.40). Then one can recognize from (Eq.41) and (Eq.42) that even mode current and odd mode current are the eigenvectors of the symmetric impedance matrix. Likewise even mode voltage and odd mode voltage are the eigenvectors of the associated admittance matrix.

\[
\begin{align*}
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix} &= \begin{bmatrix}
Z_a & Z_b \\
Z_b & Z_a
\end{bmatrix} \quad \text{(Eq.40)}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
Z_a & Z_0 \\
Z_0 & Z_a
\end{bmatrix}^{-1} &= \begin{bmatrix}
Z_a + Z_b & 1 \\
1 & 1
\end{bmatrix} \quad \text{(Eq.41)}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
Z_a & Z_0 \\
Z_0 & Z_a
\end{bmatrix}^{-1} &= \begin{bmatrix}
Z_a - Z_b & 1 \\
1 & 1
\end{bmatrix} \quad \text{(Eq.42)}
\end{align*}
\]
3.2 Mode Conversion due to Signal Path Skew

From the discussion above, when two signal lines are not symmetric, mode conversion occurs. As an asymmetric signal path example, let’s examine non coupled dual transmission lines whose characteristic impedance are the same (50ohm for example), but their line length is different. When odd mode sinusoidal signal is sent from one end of the signal path, the outputs at the two lines at another end are expressed by (Eq. 43) and (Eq. 44). Then the differential mode signal and the common mode signal are expressed by (Eq. 45) and (Eq. 46).

\[
\begin{align*}
  v_1(t) &= \cos\left(\frac{2\pi t}{T}\right) \\
  v_2(t, t_{skew}) &= -\cos\left[\frac{2\pi (t + t_{skew})}{T}\right]
\end{align*}
\]  
(Eq. 43)

\[
\begin{align*}
  v_d(t, t_{skew}) &= v_1(t) - v_2(t, t_{skew}) \\
  v_c(t, t_{skew}) &= \frac{v_1(t) + v_2(t, t_{skew})}{2}
\end{align*}
\]  
(Eq. 44)

(Eq. 45)

(Eq. 46)

With the sinusoidal signals’ period normalized to 1 and the skew of 10% of the signal period, the two single-ended signals are shown in Fig. 7. The corresponding differential and common mode signals are shown in Fig. 8. Note that the common mode signal does not exist with zero skew. It is the skew that converts part of the original differential signal energy into common mode energy.

![Fig.7 Skewed Signals at Two Output Ports](image)

![Fig.8 Differential and Common Modes Signals at Output Ports](image)
Normalizing each line impedance to 1 as expressed by (Eq.47), the differential signal power as the function of the skew is expressed by (Eq.48) and the common mode power is expressed by (Eq.49).

\[ Z_0 = \frac{Z_d}{2} = 2Z_c = 1 \]  \hspace{1cm} \text{(Eq.47)}

\[ P_d(t, t_{skew}) = \left( \frac{v_d(t, t_{skew})}{Z_d} \right)^2 \]
\[ = \frac{1}{2} \times \frac{1}{T_0} \int_0^T v_d(t, t_{skew})^2 \, dt \]  \hspace{1cm} \text{(Eq.48)}

\[ P_c(t, t_{skew}) = \left( \frac{v_c(t, t_{skew})}{Z_c} \right)^2 \]
\[ = 2 \times \frac{1}{T_0} \int_0^T v_c(t, t_{skew})^2 \, dt \]  \hspace{1cm} \text{(Eq.49)}

The calculated differential and common modes power is shown in Fig.9 with the signal path skew varying from 0% to 100% of the sinusoidal signal period. Since \( v_1 \) and \( v_2 \) without signal path skew are the already 50% skewed sinusoidal signals, additional 50% skew due to the signal path completely eliminates the originally intended differential mode.

![Fig.9 Differential and Common Modes Power as the Function of Skew](image)

We have discussed how the differential and the common modes energy varies depending on the skew between the two sinusoidal signals of a given frequency. This also means that the amount of the differential and the common modes energy varies depending on the frequency of the two sinusoidal signals of a given absolute skew. Normalizing the line impedance to 1 as done for (Eq.48) and (Eq.49), the differential signal power as the function of the frequency is expressed by (Eq.50), and the common mode power is expressed by (Eq.51). Let’s assume the same transmission lines as discussed in Fig.7 and Fig.8, where their characteristic impedance is the same, but their line length is different by 0.1 second. The calculated differential and common modes power is shown in Fig.10 with the signal frequency varying from 0.1Hz to 10Hz.
3.3 Bandwidth Reduction due to Signal Path Skew

The frequency domain information such as the one shown in Fig.10 is critical for our applications because high speed digital signal has broad frequency spectrum. Fig.10 indicates that skewed signal path intended for differential signal transmission behaves as a low pass filter even if each signal line does not cause signal loss such as dielectric loss and skin effect loss. Using two skewed single-ended signals expressed by (Eq.43) and (Eq.44), the corresponding differential signal can be obtained as expressed by (Eq.52). Thus, the cut of frequency of this signal path for differential signal can be obtained as a function of the skew by solving (Eq.52). The -3dB bandwidth and -1dB bandwidth vs. the dual lossless transmission lines skew is plotted in Fig.11. For example, 25ps skew makes the -3dB bandwidth of a differential signal path only 10GHz although each transmission line has no loss.

\[
f = \frac{1}{T}
\]

\[
P_{d_{dB}}(f) \equiv 20 \times \log \left[ \frac{1}{2} \times \int_{0}^{T} v_{d}(t, t_{skew})^2 dt \right] \quad \text{ (Eq.50)}
\]

\[
P_{c_{dB}}(f) \equiv 20 \times \log \left[ 2 \times \int_{0}^{T} v_{c}(t, t_{skew})^2 dt \right] \quad \text{ (Eq.51)}
\]

![Graph showing differential and common modes power as a function of frequency](image)

Fig.10 Differential and Common Modes Power as the Function of Frequency

\[
v_1(t) = \cos \left( \frac{2\pi t}{T} \right)
\]

\[
v_2(t, t_{skew}) = -\cos \left[ 2\pi \left( t + \frac{t_{skew}}{T} \right) \right]
\]

\[
v_d(t, t_{skew}) = v_1(t) - v_2(t, t_{skew}) = 2 \cdot \cos \left( 2\pi \left( t + \frac{t_{skew}}{2} \right) \right) \cdot \cos (\sigma t_{skew})
\]

\[
v_d(t, t_{skew}) = v_d(t, f, t_{skew}) \quad \text{ (Eq.52)}
\]
Fig. 11 Skew of Dual Transmission Lines vs. Differential Signal Path Bandwidth